Remarks

Reconsideration of the application is requested in view of the amendments made in the claims and the remarks appearing below.

1. With regard to independent claims 1 and 35, the examiner has stated that since the claims recite a reduced resolution signal and a filtered reduced resolution signal it is uncertain which of these is to undergo interpolation in accordance with the invention. The examiner also stated that since the specification teaches that it is the filtered reduced resolution signal which is interpolated, the claims are being examined with this understanding.

The examiner is correct in interpreting that it is the filtered reduced resolution signal which is interpolated. Claims 1 and 35 have been amended to specifically recite this fact. Accordingly, it is believed that any confusion which had existed with respect to the claimed subject matter has been eliminated.

Claim 39 has also been amended to specifically recite that linear low pass filtering is performed on the up-sampled filtered signal to produce the digital output signal.

2. Claims 1 - 6, 11, 12, 13 - 25, 35 - 40 and 45 - 49 have been rejected under the first paragraph of 35 U.S.C. § 112 as failing to comply with the enablement requirement. In support of this rejection the examiner has alleged that the claims contain subject matter which is not described in the specification in such a way as to enable those skilled in the art to make and/or use the invention.

The same claims have also been rejected a second time under the first paragraph of 35 U.S.C. § 112. In support of the rejection the examiner has asserted that the specification, "... while being enabling for performing resolution reduction filtering and performing interpolation on a median filtered signal, does not reasonably provide enablement for reducing a resolution of a signal as well as performing interpolation on a filtered reduced resolution signal."

The examiner has also asserted that the process of resolution reduction and interpolation are performed by linear low pass filtering prior to down-sampling for the resolution reduction and linear low pass filtering following up-sampling for the interpolation step. The examiner has concluded that those skilled in the art would have to perform undue experimentation in order to be able to practice the subject matter recited in the rejected claims since resolution reduction and interpolation are known image processing techniques that can be performed without linear low pass filtering.

It is believed that the examiner intended to assert one ground of rejection, particularly since the first statement of rejection, although having a conclusory statement, does not include any specific reasons for the rejection. Thus, applicant's response will be made as though one rejection was intended although the arguments made in this paper will also be responsive should the rejections be intended to be separate.

Applicant traverses these grounds of rejection. The subject matter of claims 1 - 6, 11, 12, 13 - 25, 35 - 40 and 45 - 49 is fully supported by the application as filed and the claims comply with the requirements for patentability established by 35 U.S.C. § 112.

It is submitted that those skilled in the art, having the benefit of applicant's extensive and detailed disclosure in the present application, in conjunction with their knowledge of the state of the art at the time the application was filed, would be able to practice, without any undue experimentation, the claimed subject matter in a scope commensurate with that of the claims.

The method of applicant, as recited in claim 1, includes the step of interpolating a filtered reduced signal to provide a digital output signal. As described in the specification, and as recited in claim 5, in a preferred embodiment interpolation of the signal can be provided in a two step process. In this embodiment, in the first step the filtered reduced signal is up-sampled to produce an up-sampled filtered signal and the second

step performs linear low pass filtering of the up-sampled filtered signal to provide the digital output signal.

Generally speaking, interpolation of a digital signal involves adding data samples in-between the existing samples in the digital signal. These new samples could be generated by taking a weighted combination of the existing samples in the neighborhood of the new sample. The examiner has implied that this method is well known. Another method to accomplish the same result is to first add samples with value 0 in-between the existing samples (commonly called "up-sampling") followed by filtering the resulting up-sampled digital signal with a low-pass filter. This two-part procedure for interpolation is very useful when it is desired to perform a frequency analysis of the interpolation operation.

This two-part interpolation procedure is well known in the art. See Multirate Systems and Filter Banks, P.P. Vaidyanathan, Prentice Hall P T R, Englewood Cliffs, NJ 07632, Copyright 1993. Enclosed are copies of pages 100-109.

Similarly, the operation of resolution reduction can also be separated into two steps, namely low-pass filtering that removes high frequencies that can not be represented in the reduced resolution domain followed by down-sampling that discards intermediate samples of the digital signal. This procedure is also well known. See, in the reference cited above, the Section on decimation and interpolation filters on page 105 and in particular, Figures 4.1-7 and 4.1-8 on page 107.

In summary, it has been shown that the claimed subject matter satisfies the criteria for patentability established by 35 U.S.C. § 112. Reconsideration of these grounds of rejection and withdrawal thereof are respectfully requested.

3. Claims 1 -6, 11, 12, 13 - 19, 30 - 40 and 45 - 49 have been rejected under the second paragraph of 35 U.S.C. § 112 as being indefinite.

A. With regard to claims 1 and 35, as discussed above, the amendments made in these claims are believed to overcome the rejection. The claims now specifically recite that interpolation is performed on the filtered reduced resolution signal.

B. The examiner has stated that claim 30 has an improper dependency which makes it indefinite. Two reasons have been given in support of this ground of rejection. The first reason asserted by the examiner is that the preamble of claim 30 recites a multi-resolution filtering system which is dependent from a method claim. The second reason is that claim 30 incorporates claim 18 which is a dependent claim with the result that the examiner is unsure whether just claim 18 is incorporated into claim 30 or also some or all of the claims from which claim 18 depends.

Claim 30 has now been amended to incorporate claim 20 which is an independent claim and recites a multi-resolution filter. It is believed that this amendment overcomes this ground of rejection.

C. The examiner has asserted that there is insufficient basis in claims 5 and 39 for "low pass filtering of the filtered signal". Claim 39, has been amended, to recite "low-pass filtering of the up-sampled filtered signal". With respect to claim 5 which is dependent on claim 1, the amendment made in claim 1 has removed any ambiguity which may have existed. Claims 5 and 39 now particularly point out and distinctly claim the subject matter recited therein.

In summary, the claims, as amended, have been shown to meet the requirements of 35 U.S.C. § 112. Reconsideration of these grounds of rejection and withdrawal thereof are respectfully requested.

4. Claims 1. 2, 11, 12, 20, 21, 35 and 35 have been rejected under 35 U.S.C. § 102(b) as being anticipated by U.S. Patent 5,528,301 ("Hau et al"). Applicant traverses this ground of rejection. The purpose of the method of Hau et al is very different from the purpose of applicant's method. The reference does not teach each and every limitation of the subject matter recited in the claims.

The claimed method and apparatus of applicant are directed to the selective attenuation of corruption in a digital signal. Corruption can be introduced into a digital signal by the phenomenon known as "aliasing". For a discussion of this phenomenon see page 4, lines 21 -25. In particular, applicant's method is designed to attenuate aliasing artifacts that occur in digital images captured with a sensor that does not have a suitable

anti-aliasing filter. See the discussion extending from page 5, line 12 to page 6, line 5. In applicant's method the output signal is devoid of any aliasing artifacts which were already present in the input signal.

In the method of applicant the resolution of the digital input signal is first reduced to provide a reduced resolution signal that has fewer data samples, or points, than the input signal. The reduced resolution signal is then median filtered, i.e., for any one sample in the reduced resolution signal the median filter computes the median of its neighboring *n* samples and replaces the sample value with the median value, to provide a filtered reduced resolution signal. The filtered reduced resolution signal is then interpolated, that is, the number of data points is increased, to provide the digital output signal.

The process of decimation and interpolation reduces the computational complexity involved in the claimed method. See, for example, the discussion on page 17, lines 3 - 14.

On the other hand, the method of Hau et al is aimed at video conversion from one format into another. In contrast to applicant's method, the method of Hau et al is primarily concerned with not introducing aliasing artifacts in the conversion process rather than eliminating pre-existing aliasing artifacts.

The bandlimiting filter used in the Hau et al method is used when reducing the sampling rate from the input format to the output format. The role of the bandlimiting filter is to remove any high frequencies that are present in the input signal and that can not be represented in the lower sampling rate of the output filter. To achieve this result, the bandlimiting filter is necessarily a linear filter since all frequencies outside the cutoff range have to be attenuated by this filter independent of the nature of the input signal. In particular, Hau et al teaches using a finite impulse response ("FIR") filter for this purpose. See column 6, lines 20-24.

The linear bandlimiting filter is quite unlike the median filter utilized by applicant which is a non-linear filter. The non-linearity of the median filter precludes the analysis of this type of filter in the frequency domain. The median filter switches its behavior depending on the input signal. For instance, if a median filter is presented with a step-edge, as illustrated in Fig. 10 of the present application, the output signal is also a step-edge with no degradation. Since all the frequencies in the input signal are reproduced in the output signal the median filter in this case is not acting as a bandlimiting filter and can not be considered as such for the purpose of asserting that the Hau et al reference supports an anticipation rejection of the claims.

Indeed, the method of Hau et al could not achieve its desired result of anti-aliasing (see, column 3, lines 41-46) if a median filter were to be substituted for the

specified bandlimiting filter since the median filter would not attenuate the high frequencies in some cases thus leading to aliasing in the output signal. Conversely, if a linear bandlimiting filter were to be substituted for the required median filter in the method and apparatus of applicant, the desired result of attenuating aliasing artifacts in the input signal without degrading the original signal could not be achieved.

In view of the foregoing it is apparent that Hau et al does not teach each and every element of the presently claimed subject matter and therefore does not properly support a rejection under 35 U.S.C. § 102. Reconsideration of this ground of rejection and withdrawal thereof are respectfully requested.

5. Claims 3, 22 and 37 have been rejected under 35 U.S.C. § 103(a) as being unpatentable over Hau et al in view of U.S. Patent 6,519,288 ("Vetro et al").

Applicant traverses this ground of rejection. These claims are each dependent upon a claim discussed above with respect to the primary reference and are directed to an embodiment of the invention wherein the reducing of the resolution of the digital input signal involves linear low pass filtering of the signal. It has been shown that the primary reference does not teach or in any way suggest the advantageous method, apparatus or multi-resolution filter of applicant.

The disclosure of Vetro et al. does not render the rejection any more effective. As is the case with Hau et al, the secondary reference is not directed to removing aliasing artifacts from a digital input signal. Those skilled in the art and knowing of the disclosures of these references would not be placed in possession of the presently claimed subject matter.

Reconsideration of this ground of rejection and withdrawal thereof are respectfully requested.

6. Claims 4, 23 and 38 have been rejected under 35 U.S.C. § 103(a) as being unpatentable over Hau et al in view of Vetro et al and further in view of U.S. Patent 5,831,677 ("Streater et al").

Applicant traverses this ground of rejection. These claims are each dependent upon a claim discussed above with respect to the primary reference and are directed to an embodiment of the invention wherein the reducing of the resolution of the digital input signal involves mean filtering of the digital input signal. It has been shown above that Hau et al and Vetro et al, viewed individually or together do not teach or in any way suggest the advantageous method, apparatus or multi-resolution filter of applicant.

The disclosure of Streater et al does not render the rejection any more effective. As is the case with Hau et al and Vetro et al, Streater et al is not directed to removing aliasing artifacts from a digital input signal. Those skilled in the art and knowing of the disclosures of these references would not be placed in possession of the presently claimed subject matter.

Reconsideration of this ground of rejection and withdrawal thereof are respectfully requested.

7. Claims 5, 6, 24, 25, 39 and 40 have been rejected under 35 U.S.C.§ 103(a) as being unpatentable over Hau et al in view of U.S. Patent 5,844,617 ("Faroudja et al").

Applicant traverses this ground of rejection. These claims are each dependent upon a claim discussed in detail above with respect to the primary reference and are directed to embodiments of the invention wherein the interpolation step is carried out in a particular manner. It has been shown above that Hau et al does not teach or in any way suggest the advantageous method, apparatus or multi-resolution filter of applicant.

The disclosure of Faroudja et al et al does not render the rejection any more effective. As is the case with the other references discussed above, Faroudja et al is not directed to removing aliasing artifacts from a digital input signal.

Those skilled in the art and knowing of the disclosures of these references would not be placed in possession of the presently claimed subject matter.

Reconsideration of this ground of rejection and withdrawal thereof are respectfully requested.

8. Claims 7 - 10, 26 -29 and 41 - 44 have been rejected under 35 U.S.C.§ 103(a) as being unpatentable over Hau et al in view of Vetro et al and Faroudja et al.

Applicant traverses this ground of rejection. These claims are directed to particular embodiments of applicant's invention. These embodiments involve median filtering a reduced resolution signal in a particular manner. It has been shown above that these references do not teach or in any way suggest the advantageous method, apparatus or multi-resolution filter of applicant.

Those skilled in the art and knowing of the disclosures of these references would not be placed in possession of the presently claimed subject matter.

Reconsideration of this ground of rejection and withdrawal thereof are respectfully requested.

9. Claims 13 - 19, 30 -34 and 45 - 49 have been rejected under 35 U.S.C. § 103(a) as being unpatentable over U.S. Patent 5,841,480 ("Rhodes") and Hau et al.

Applicant traverses this ground of rejection. These claims are directed to embodiments of the invention wherein a second digital signal is produced from a first digital signal which includes a first luminance signal, a first chrominance signal and a second chrominance signal. The first and second chrominance signals are each filtered according to the method of claim 1 to produce,

respectively, first and second filtered chrominance signals.

It has been shown above that Hau et al does not teach or in any way suggest the advantageous method, recited in claim 1. Rhodes, does teach the conversion of a digital signal to luminance space but, as is the case with Hau et al, does not teach or suggest removing aliasing artifacts from a digital signal.

Those skilled in the art and knowing of the disclosures of these references would not be placed in possession of the presently claimed subject matter.

Reconsideration of this ground of rejection and withdrawal thereof are respectfully requested.

In summary it has been shown that the claims, as amended, are proper in form for allowance and in substance are directed to subject matter which is wholly novel and unobvious over the references of record. Reconsideration of the application and allowance of the claims are respectfully requested.

Respectfully submitted,

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CERTIFICATE OF MAILING

I hereby certify that this paper (along with any paper referred to as being attached or enclosed) is being deposited with the United States Postal Service on the date shown below with sufficient postage as first class mail in an envelope addressed to the Commissioner for Patents, P.O. Box 1450, Alexandria, VA 22313-1450.

Date: January 26, 2004

Gaetano D. Maccarone
Registration No. 25,173

4

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Fundamentals of Multirate Systems

4.0 INTRODUCTION

This chapter is basic to the study of multirate systems and filter banks. Section 4.1 introduces decimation, interpolation, and filter bank systems, and Sec. 4.2 discusses interconnections of building blocks. The polyphase decomposition is introduced in Sec. 4.3, along with some applications. Multistage filter design is discussed in Sec. 4.4. Several applications of multirate systems are described in Sec. 4.5. Many special types of filters such as half-band filters and Nyquist filters, and complementary filter banks are discussed in Sec. 4.6. Finally, Sec. 4.7 introduces *multigrid techniques* which are well known in the literature on numerical computation.

Some of these topics have also been covered in various chapters of Crochiere and Rabiner [1983]. However, a number of new topics are also introduced here, for example, complementary filters (power complementary, Euclidean complementary, etc.), and multigrid methods.

4.1 BASIC MULTIRATE OPERATIONS

4.1.1 Decimation and Interpolation

The most basic operations in multirate digital signal processing are decimation and interpolation. In order to describe these, two new building blocks are introduced, called the *decimator* and the *expander*.

The M-fold decimator. Figure 4.1-1(a) shows the M-fold decimator, which takes an input sequence $x(n)$ and produces the output sequence

$$y_D(n) = x(Mn), \tag{4.1.1}$$

where M is an integer. Only those samples of $x(n)$ which occur at time equal to multiples of M are retained by the decimator. Figure 4.1-2 demonstrates the idea for $M = 2$. The decimator is also called a *downsampler*,

subsampler, sampling rate compressor, or merely a *compressor*. We will use the term "decimator" consistently. As will be mathematically substantiated, decimation results in aliasing unless $x(n)$ is bandlimited in a certain way. In general, therefore, it may not be possible to recover $x(n)$ from $y_D(n)$ because of loss of information.

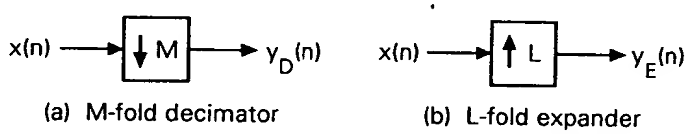

Figure 4.1-1 The decimator and expander.

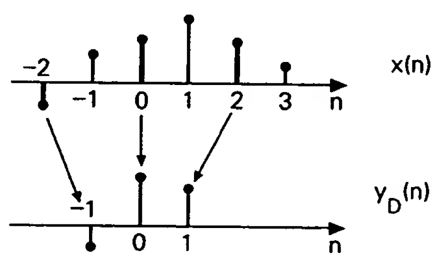

Figure 4.1-2 Demonstration of decimation for $M = 2$. The samples of $x(n)$ shown by heavy lines are retained.

The L-fold expander. Figure 4.1-1(b) shows a building block which is commonly called an *L-fold expander*. This device takes an input $x(n)$ and produces an output sequence

$$y_E(n) = \begin{cases} x(n/L), & \text{if } n \text{ is integer-multiple of } L \\ 0, & \text{otherwise.} \end{cases} \tag{4.1.2}$$

Here L is an integer. Figure 4.1-3 is a demonstration of this operation for $L = 2$. It is evident that the expander does not cause loss of information. We can recover the input $x(n)$ from $y_E(n)$ by L-fold decimation.

Other names for the expander are: sampling rate expander, upsampler, and interpolator. Of these, the term 'interpolator' is really a misnomer. We will consistently use the term 'expander' in this text. The expander is used in interpolation, but a filter is required to complete the process; we will see how the zero-valued samples are converted into interpolated samples by using a lowpass filter at the output of the expander.

Transform Domain Analysis of Decimators and Expanders

First consider the expander which is easier to analyze. We have

$$\begin{aligned}
Y_E(z) &= \sum_{n=-\infty}^{\infty} y_E(n)z^{-n} = \sum_{n=\text{mul. of } L} y_E(n)z^{-n} \\
&= \sum_{k=-\infty}^{\infty} y_E(kL)z^{-kL} = \sum_{k=-\infty}^{\infty} x(k)z^{-kL} \quad \text{from (4.1.2)} \\
&= X(z^L).
\end{aligned} \tag{4.1.3}$$

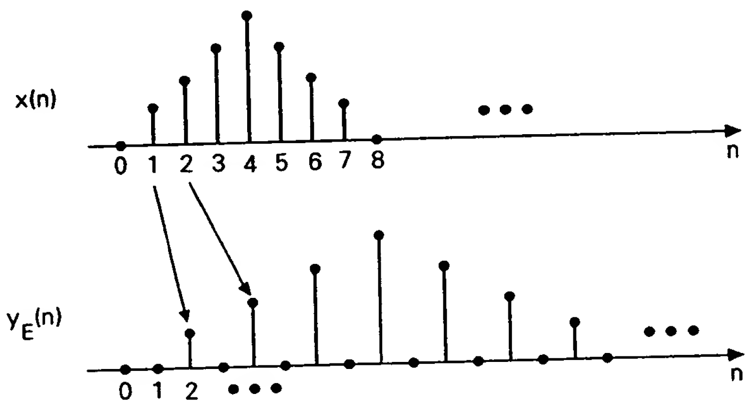

Figure 4.1-3 Demonstration of the expander for $L = 2$.

So $Y_E(e^{j\omega}) = X(e^{j\omega L})$. This means that $Y_E(e^{j\omega})$ is an L-fold compressed version of $X(e^{j\omega})$ as demonstrated in Figs. 4.1-4(a),(b). The multiple copies of the compressed spectrum are called *images,* and we say that the expander creates an imaging effect. [The quantity $X(e^{j\omega})$ in the figure is taken to be nonsymmetric with respect to $\omega = 0$, to improve clarity and generality.]

For the M-fold decimator (4.1.1), we now derive an expression for the output $Y_D(e^{j\omega})$ in terms of $X(e^{j\omega})$. We will show that

$$Y_D(e^{j\omega}) = \frac{1}{M}\sum_{k=0}^{M-1} X(e^{j(\omega-2\pi k)/M}). \tag{4.1.4}$$

This can be graphically interpreted as follows: (a) stretch $X(e^{j\omega})$ by a factor M to obtain $X(e^{j\omega/M})$, (b) create $M-1$ copies of this stretched version by shifting it uniformly in successive amounts of 2π, and (c) add all these shifted stretched versions to the unshifted stretched version $X(e^{j\omega/M})$, and divide by M. The stretched quantity $X(e^{j\omega/M})$ does not have period 2π, but after adding the shifted versions the result is periodic with period 2π (which is a

requirement for the Fourier transform of a sequence). See Figs. 4.1-4(c) and 4.1-5 which demonstrate these for $M = 2$, and 3.

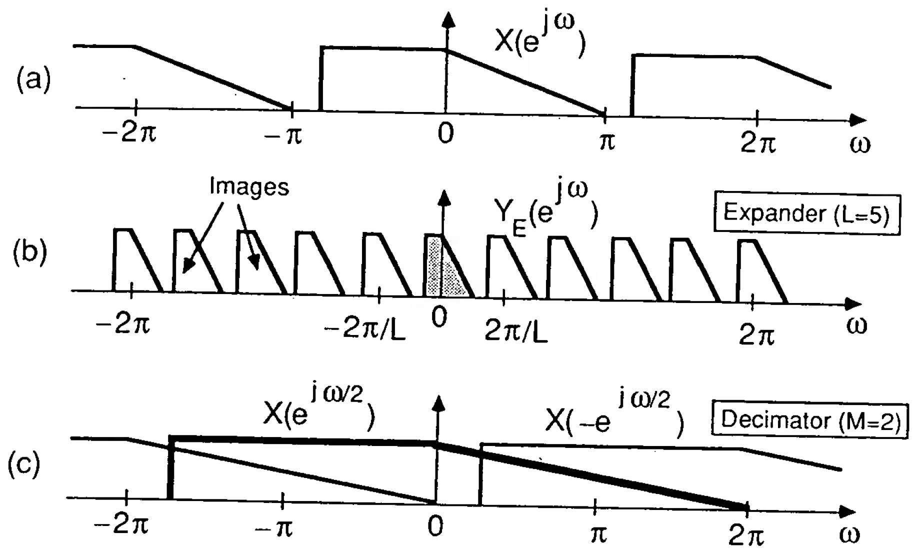

Figure 4.1-4 Transform-domain effects of the expander and decimator. The Fourier transforms of (a) the input signal $x(n)$, (b) the expanded signal ($L = 5$), and (c) the decimated signal ($M = 2$).

Proof of (4.1.4). The z-transform of $y_D(n)$ can be written as

$$Y_D(z) = \sum_{n=-\infty}^{\infty} y_D(n)z^{-n} = \sum_{n=-\infty}^{\infty} x(Mn)z^{-n}.$$

Define an intermediate sequence

$$x_1(n) = \begin{cases} x(n), & n = \text{mul. of } M \\ 0, & \text{otherwise} \end{cases} \tag{4.1.5}$$

so that $y_D(n) = x(Mn) = x_1(Mn)$. Now

$$Y_D(z) = \sum_{n=-\infty}^{\infty} x_1(Mn)z^{-n} = \sum_{k=-\infty}^{\infty} x_1(k)z^{-k/M}. \tag{4.1.6}$$

This step is valid because $x_1(k)$ is zero unless k is a multiple of M. So

$$Y_D(z) = X_1(z^{1/M}). \tag{4.1.7}$$

It only remains to express $X_1(z)$ in terms of $X(z)$. For this note that (4.1.5) can be written as

$$x_1(n) = C_M(n)x(n) \tag{4.1.8}$$

where $C_M(n)$ is the 'comb' sequence defined as

$$C_M(n) = \begin{cases} 1, & n = \text{mul. of } M, \\ 0, & \text{otherwise.} \end{cases} \tag{4.1.9}$$

We can express the comb sequence as

$$C_M(n) = \frac{1}{M} \sum_{k=0}^{M-1} W_M^{-kn}, \tag{4.1.10}$$

where W_M is the Mth root of unity defined as

$$W_M = e^{-j2\pi/M}. \tag{4.1.11}$$

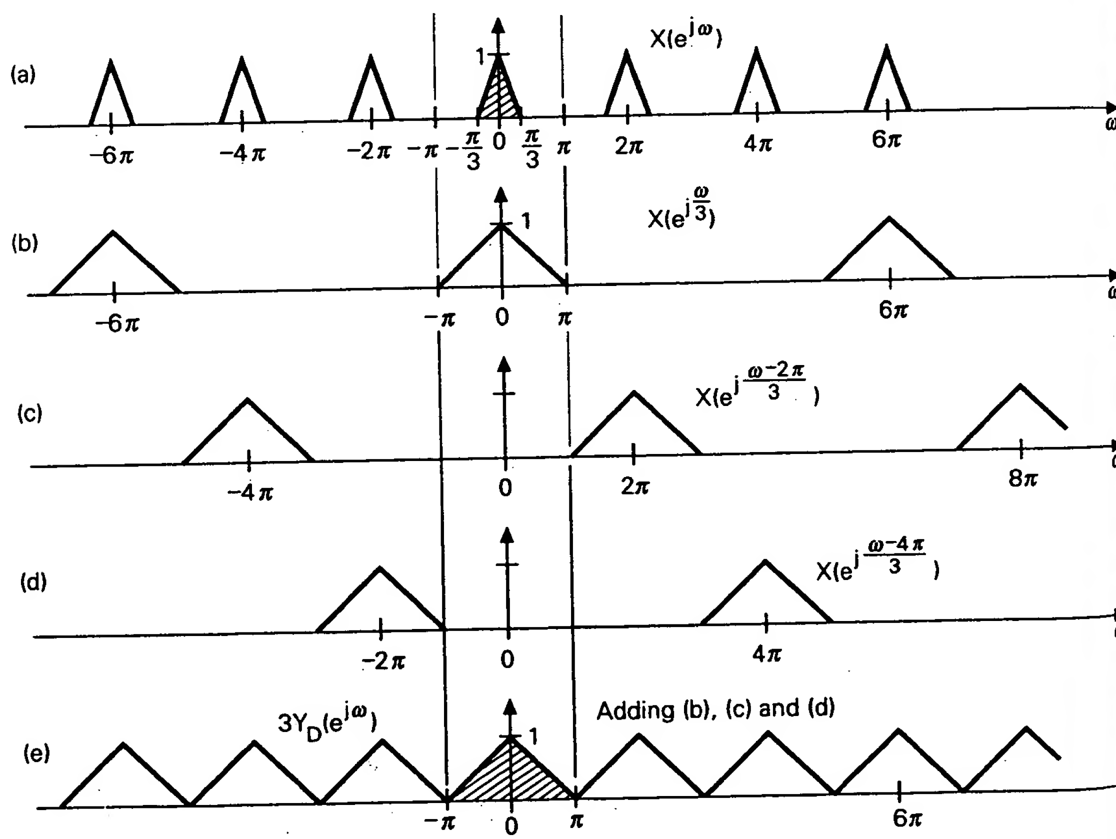

Figure 4.1-5 Demonstrating the frequency-domain effect of decimation with $M = 3$.

The subscript M on W is usually deleted, unless there is room for confusion. We can now obtain

$$X_1(z) = \frac{1}{M}\sum_{k=0}^{M-1}\sum_{n=-\infty}^{\infty} x(n)W^{-kn}z^{-n} = \frac{1}{M}\sum_{k=0}^{M-1}\sum_{n=-\infty}^{\infty} x(n)(zW^k)^{-n}. \tag{4.1.12}$$

The inner summation above is equal to $X(zW^k)$ so that from (4.1.7)

$$Y_D(z) = \frac{1}{M}\sum_{k=0}^{M-1} X(z^{1/M}W^k). \tag{4.1.13}$$

In terms of the frequency variable ω this becomes (4.1.4) indeed.

We often use the following notation to indicate the relation (4.1.13):

$$Y_D(z) = X(z)\Big|_{\downarrow M} \tag{4.1.14}$$

This notation means that $y_D(n)$ is the M-fold decimated version of $x(n)$.

Aliasing Created by Decimation

From Fig. 4.1-4(c), which demonstrates the effect of decimation for $M = 2$, we see that the stretched version $X(e^{j\omega/M})$ can in general overlap with its shifted replicas. If this happens, we cannot recover $x(n)$ from the decimated version $y_D(n)$. This overlap effect is called *aliasing*.

Avoiding aliasing. It is clear that aliasing can be avoided if $x(n)$ is a lowpass signal bandlimited to the region $|\omega| < \pi/M$. The example in Fig. 4.1-5 demonstrates this for $M = 3$. In this case we can recover $x(n)$ from the decimated version by use of an expander, followed by filtering, as demonstrated in Fig. 4.1-6. This recovery scheme works as follows: in the frequency domain, the output $V(e^{j\omega})$ of the expander is a compressed version of $Y_D(e^{j\omega})$ [part (c)]. By using a lowpass filter $H(e^{j\omega})$ [part(d)] we can therefore eliminate the images and extract the original spectrum $X(e^{j\omega})$ [part (e)].

The above condition on bandwidth is, however, not *necessary* to avoid aliasing. For example if $X(e^{j\omega})$ is zero everywhere in $0 \leq \omega < 2\pi$ except in $\omega_1 < \omega < \omega_1 + 2\pi/M$ for some ω_1, then there is no overlap between any pair of terms in (4.1.4). Also see Problem 4.3. The most general condition for alias-free decimation can be found in [Sathe and Vaidyanathan, 1993].

It can be verified (Problem 4.4) that the decimator and expander are linear but *time-varying* (LTV) systems.

Decimation Filters and Interpolation Filters.

In most applications, the decimator is preceded by a lowpass digital filter called the *decimation filter* [Fig. 4.1-7(a)]. The filter ensures that the

signal being decimated is bandlimited. The exact bandedges of the filter depend on how much aliasing is permitted. For example, in QMF banks (Chapters 5–8), a certain degree of aliasing is usually permitted because this can eventually be canceled off. The simplest form of lowpass decimation filter has magnitude response as sketched in Fig. 4.1-7(b).

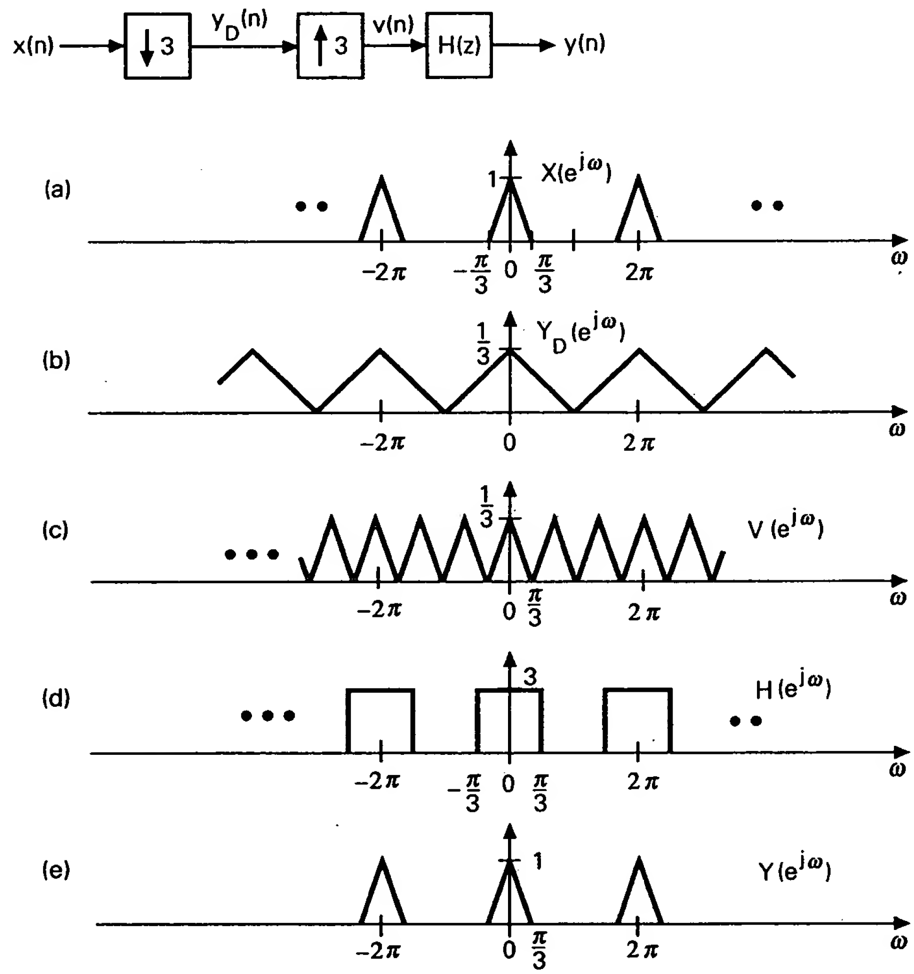

Figure 4.1-6 Recovering bandlimited $x(n)$ from its decimated version.

Next, an interpolation filter (Fig. 4.1-8) is a digital filter that *follows* an expander. The typical purpose is to suppress all the images. Thus, it retains only the shaded portion of the compressed spectrum $Y_E(e^{j\omega})$ in Fig. 4.1-4(b). Typically the interpolation filter is lowpass with cutoff frequency π/L. In the time domain, $y(n)$ is a convolution of $y_E(n)$ with the impulse response $h(n)$. The effect is that the zero-valued samples introduced by the expander are filled with 'interpolated' values [Fig. 4.1-8(c)].

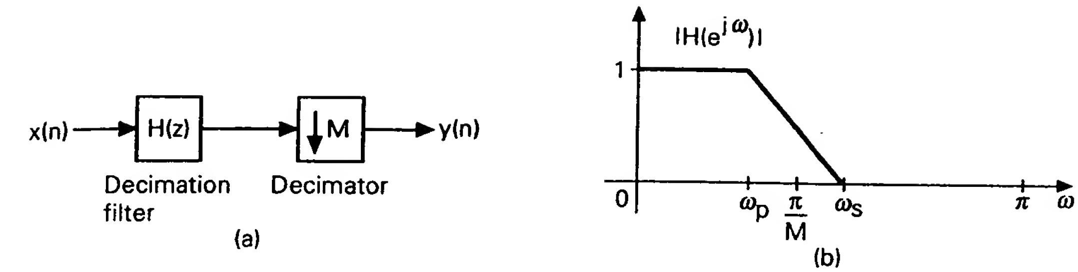

Figure 4.1-7 (a) The complete decimation circuit, and (b) typical response of the decimation filter.

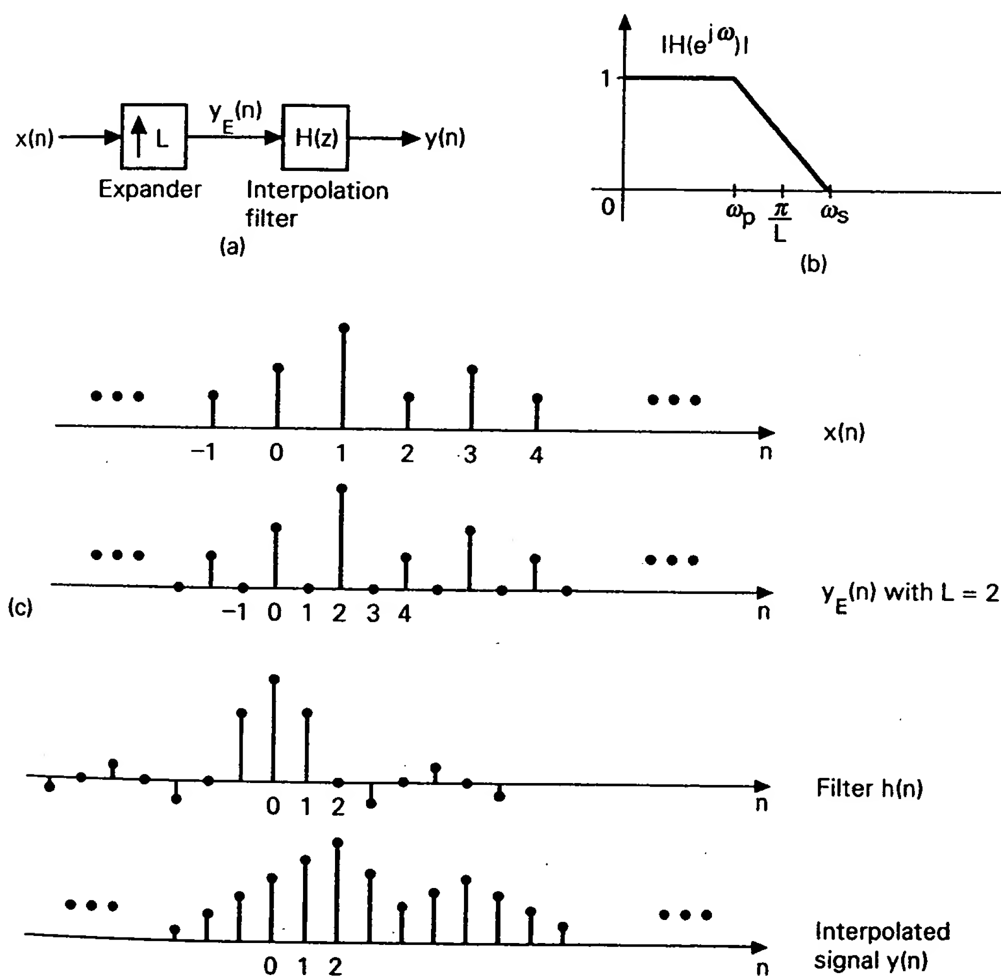

Figure 4.1-8 (a) The complete interpolation circuit, (b) typical response of the interpolation filter, and (c) examples of the sequence $x(n)$, the filter $h(n)$, and the interpolated signal $y(n)$.

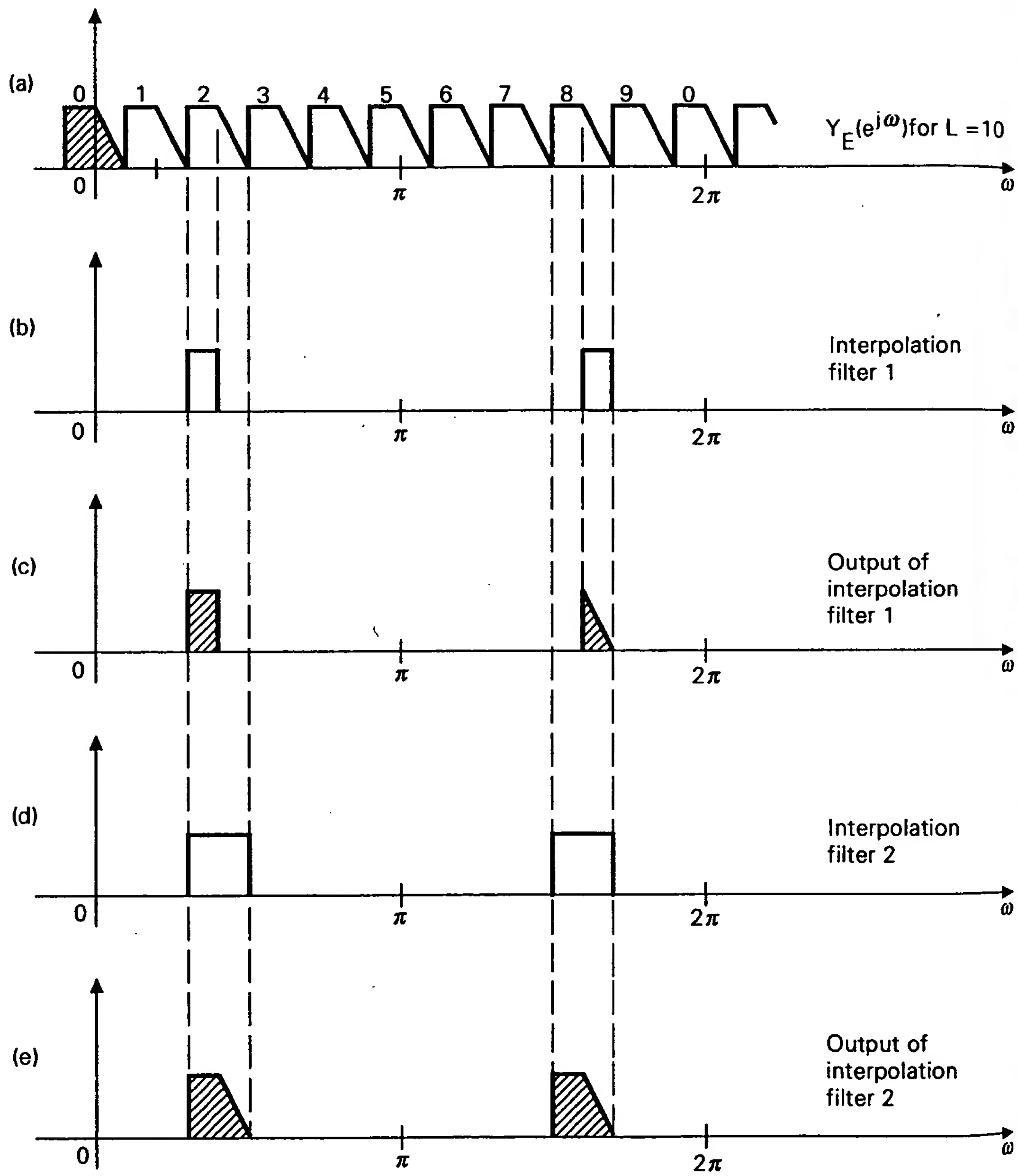

Figure 4.1-9 Demonstrating several possible choices for the interpolation filter.

More generally, it is possible to make other choices of the interpolation filter, as demonstrated in Fig. 4.1-9 for $L = 10$. Here $Y_E(e^{j\omega})$ has nine images (unshaded copies in Fig. 4.1-9(a)). If the filter is chosen as in part (b), the

filter output is as in part (c), and contains all information about $X(e^{j\omega})$ [which is the 10-fold stretched version of the shaded portion in part (a)]. If the filter is as in part (d), then two images are retained. This is analogous to cosine modulation of the shaded portion in part (a). See Problem 4.5 for precise relation between cosine modulation and interpolation-filtering. Both filtering schemes in this figure are such that the filter coefficients are real [so that the filter output is real if $x(n)$ is].

Fractional Sampling Rate Alteration

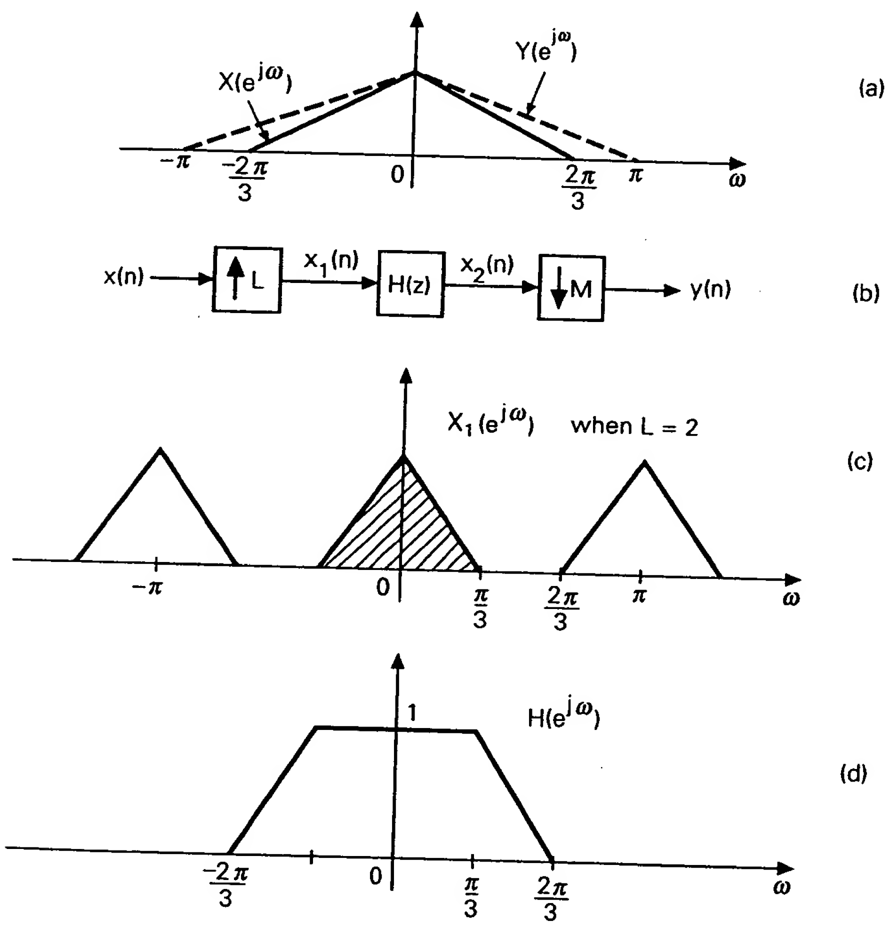

Figure 4.1-10 Pertaining to fractional decimation.

The above techniques permit us to alter the sampling rate of a signal by an integer factor (such as L or M). In some applications, however, it is necessary to change the rate by a rational fraction (such as L/M). For example consider Fig. 4.1-10(a) which shows the transform $X(e^{j\omega})$ of a signal bandlimited to $|\omega| < 2\pi/3$. We cannot decimate the signal by two because that would create aliasing error. It appears to be possible to decimate by